



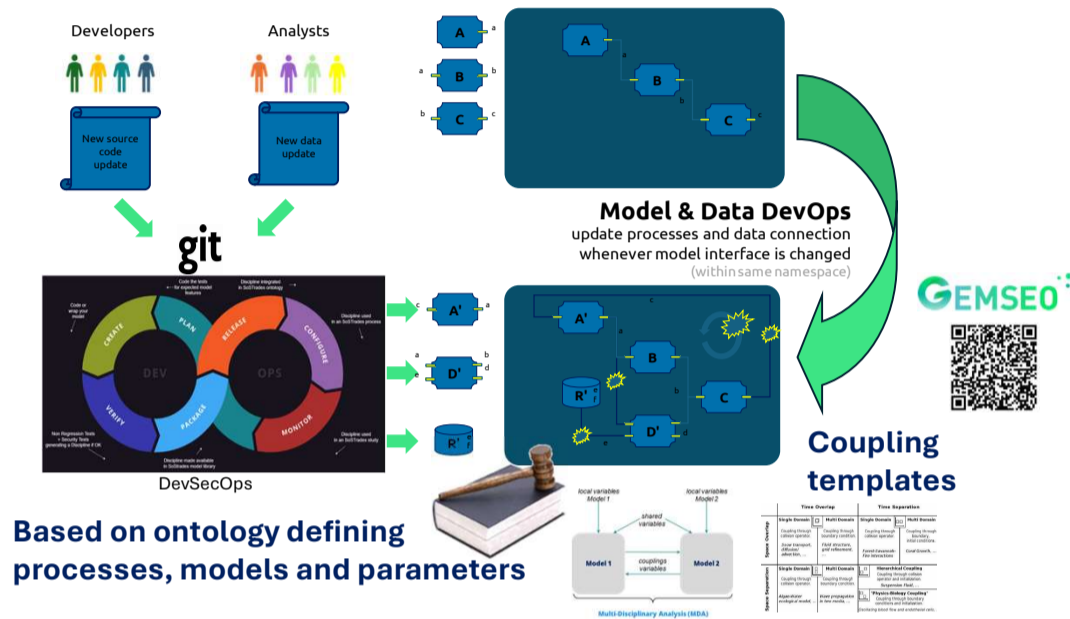
System of system approach applied to the WITNESS IAM

Matthieu Meaux, Thierry Chevalier, Marie Morere

One challenge for IAMs is capturing interactions and feedback loops between sub-models, often overlooked but crucial for accuracy. WITNESS is a recent IAM designed to address these interactions using a system of systems approach. It applies Multi-Disciplinary Analysis & Optimization (MDO) to solve the resulting complex problems.

System level modelling approach

Automated handling of interactions and couplings within system



Based on ontology defining processes, models and parameters

Fixed point theorem for system convergence unicity

Widely used in Multi-Disciplinary Analysis contexts

Definition 1. A fixed point of a mapping $T: X \rightarrow X$ of a set X into itself is an $x \in X$ which is mapped onto itself, that is $Tx = x$.

Definition 2. Let (X, d) be a metric space. A mapping $T: X \rightarrow X$ is called a contraction on X if there exists a positive constant $K < 1$ such that (1) $d(T(x), T(y)) \leq Kd(x, y)$ for all $x, y \in X$.

Theorem 2 (Banach's Fixed Point Theorem). Let (X, d) be a complete metric space and let $T: X \rightarrow X$ be a contraction on X . Then T has a unique fixed point $x \in X$ (such that $T(x) = x$).

Corollary 3 (Iterations and error bounds). The iterative sequence (2) with arbitrary $x_0 \in X$ converges (under the assumptions in Banach's Fixed Point Theorem) to the unique fixed point x of T . Error estimates are the prior estimate

$$(4) \quad d(x_m, x) \leq \frac{K^m}{1-K} d(x_0, x_1),$$

and the posterior estimate

$$(5) \quad d(x_m, x) \leq \frac{K}{1-K} d(x_{m-1}, x_m).$$

Newton method for convergence finding

fixed point iteration method in a complete normed vector space (Banach space)

$$X_{n+1} = X_n - (F'(X_n))^{-1} F(X_n)$$

with F' the Fréchet derivative of F

Means you need to define the gradient function of all your models

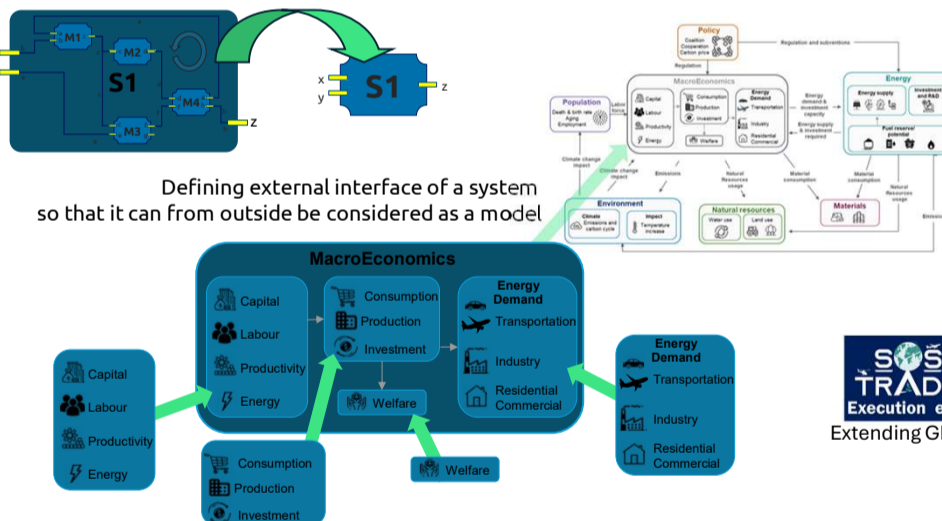
Complex step differentiation

$$f'(x) = \frac{\Im(f(x+ih))}{h} + O(h^2)$$

$f' := -1$

System of systems level modelling approach

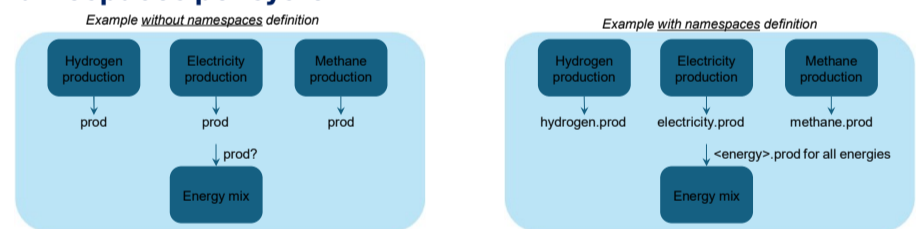
System engineering approach



Defining external interface of a system so that it can from outside be considered as a model

Automated handling of interactions and coupling between systems within uplevel system of systems

Namespaces per system



Macro-economic model specifics

Capital requires energy to be productive
 Concept of "usable capital"

$$K_u = K \frac{E_u}{E_{max}^k}$$

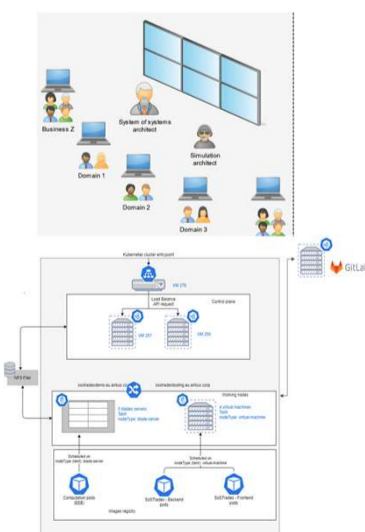
K : capital
 E_u : usable energy
 E_{max}^k : maximum energy that the capital can absorb
(based on past correlation between usable capital and usable energy production, evolves with technological progress)

then GDP becomes $Y = (1 - \Omega) \cdot A \cdot (a \cdot K u^\rho + (1 - a) \cdot L^\rho)^{\sigma/\rho}$

System of systems level usage approach

Collaborative Design Facilities inspired

- strong users & groups management
- multi-user web graphical interface
- traced parameter value changes
- traced modular validation of assumptions & results
- bulk assumptions loading / bulk results export



Scalable

- Native cloud implementation
- Automated management if execution constraints (where, which resource for which model...)
- On the fly pods allocation when studies are loaded
- Scalable proven applied mathematics algorithmics

Reference data sets handling

- Plugins to accommodate with fetching from / writing to various data repository technologies (CSV files, JSON files, RDBMS, NoSQLDB, OODB...)
- Datasets to describe data location and content
- Mappings to connect various datasets with various variables in specific namespaces
- Working for modular bulk assumptions loading / bulk results exports

Sub-systems auto-optimization

Energy subsystem auto-adaptation in generic WITNESS framework

